LEVELLING

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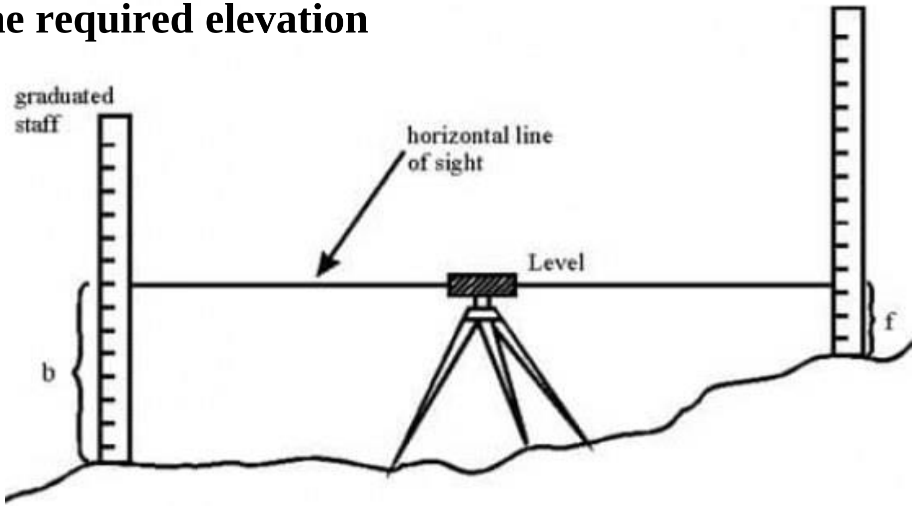
Levelling

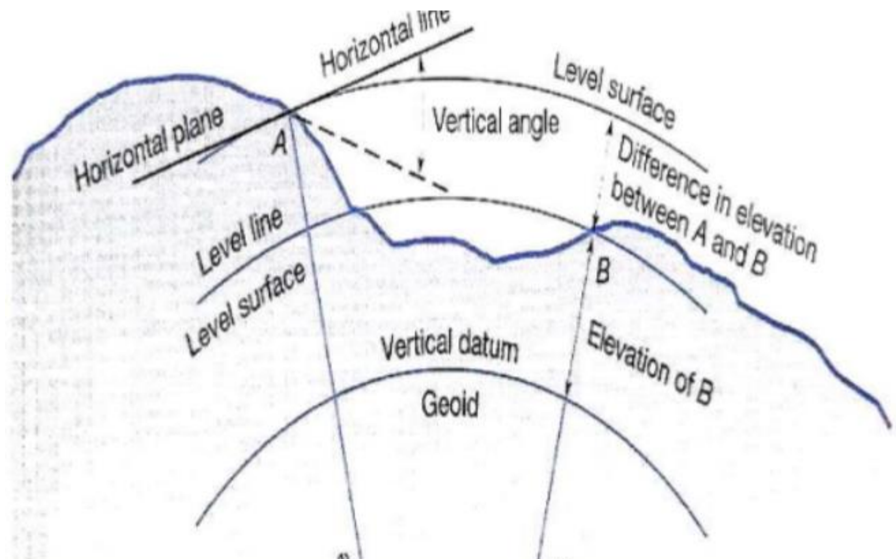
- Art of determining the relative heights of different points on, above or below the surface of earth.
- Principle – obtain horizontal line of sight with respect to which vertical distance of points above or below this line of sight are found
- Object:
  - **To find elevation of a given point with respect to a reference line (datum)**
  - **To establish points at the required elevation**





Terminology

1. Level Surface : surface parallel to mean spheroidal surface of earth
2. Level line : line lying on level surface
3. Horizontal plane : plane tangential to level surface
4. Horizontal line : line tangential to level line
5. Datum : arbitrary level surface from which elevations of points may be referred. Current datum – Bombay High
6. Mean Sea Level : average height of sea for all stages of tide obtained by averaging hourly tide over a period of 19 years





Terminology

7. Elevation or Reduced level (R.L) : height or depth of any point above or below any datum

8. Bench Mark (BM) : A fixed reference point of known elevation

9. Line of collimation: line joining the intersection of cross hairs of diaphragm to the optical centre of object glass and its continuation.

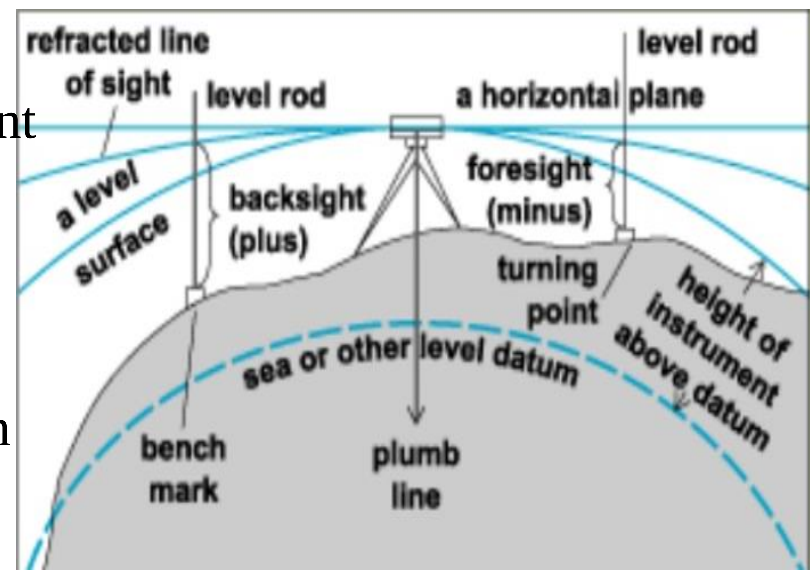
10. Height of instrument: elevation of line of collimation with respect to datum

11. Backsight (BS): staff reading taken to a point of known elevation. It is the first reading taken after the setup

12. Foresight (FS) : last staff reading before shifting instrument

13. Intermediate sight (IS): Staff reading taken on a point whose elevation is to be found

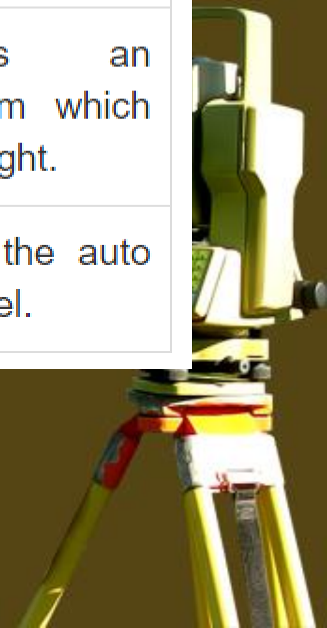
14. Change Point (CP) : point on which both BS & FS is taken

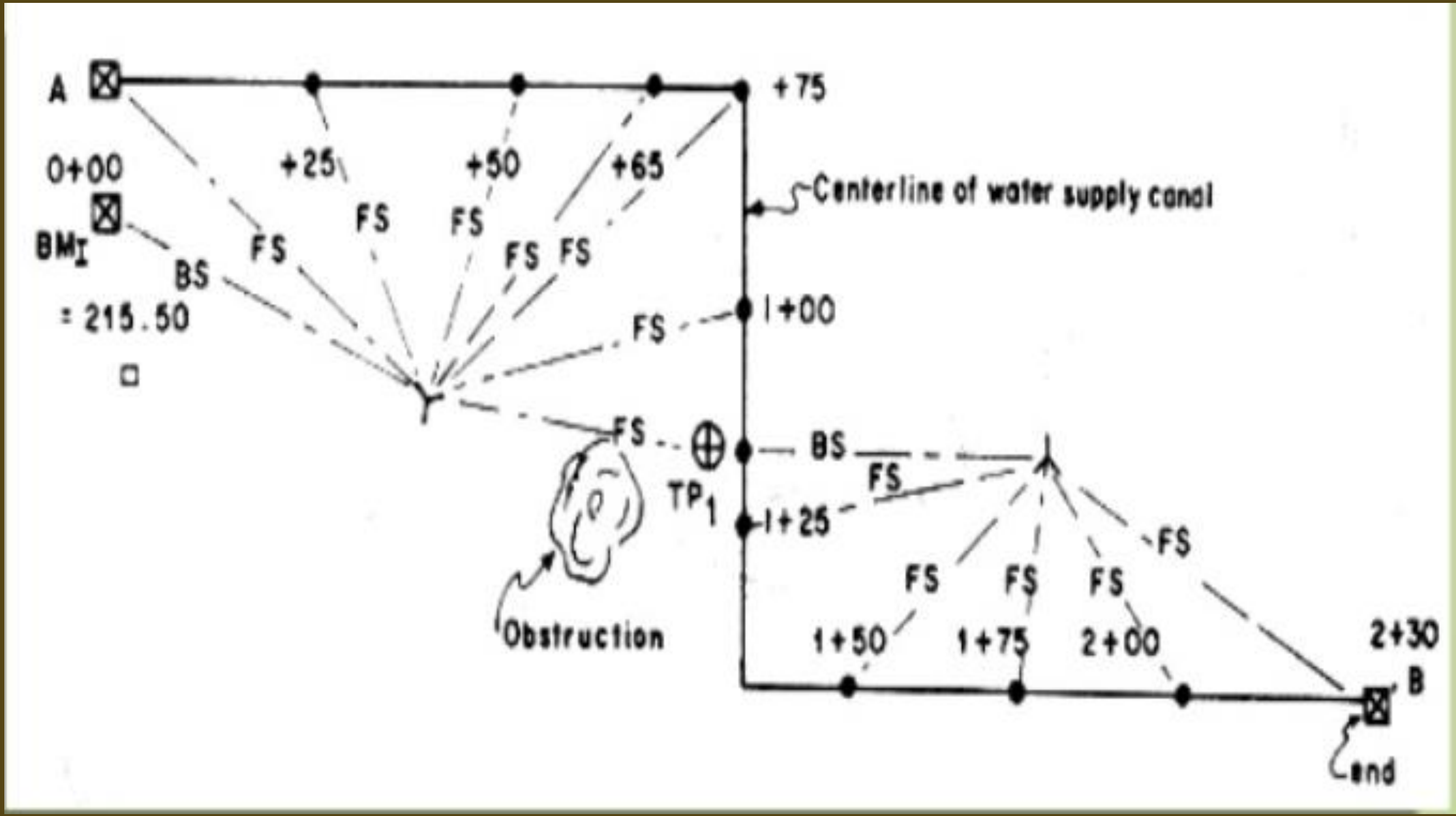


Instruments for levelling

DIFFERENCE BETWEEN DUMPY & AUTO LEVELS

	Dumpy Level	Auto Level
1	In the dumpy level survey, staff reading need to be adjusted as inverted level staff reading is seen in the eyepiece.	In the auto level, no adjustment for staff reading is required as the actual reading is seen from the eyepiece.
2	In the dumpy level, to level the bubble, one has to keep the bubble parallel to two leveling screws and then right angle to the third screw.	In the auto level, the bubble can be adjusted from any side and any angle with any 3 screws available.
3	The line of sight is manually adjusted in the dumpy level.	The auto level has an internal compensator mechanism which automatically adjusts the line of sight.
4	It is difficult to make an accurate measurement with the dumpy level.	The measurement accuracy of the auto level is higher than the dumpy level.




A
0+00
+25
+50
+65
+75
BM
= 215.50
BS
FS
Centerline of water supply canal
1+00
TP1
1+25
Obstruction
1+50
1+75
2+00
2+30
B
end

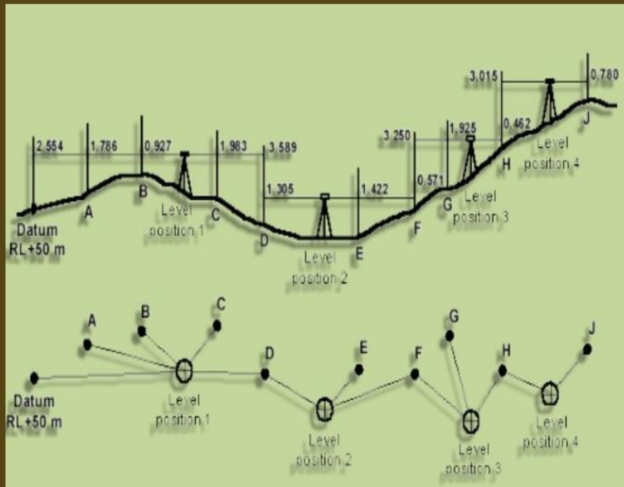
Bench Marks

1. GTS (Great Trigonometrically Survey) Bench Mark : established with high precision at regular intervals by SoI
2. Permanent Bench Mark : fixed in between GTS by govt. agencies such as PWD eg: milestones
3. Arbitrary Bench Mark: reference points whose R.L is assumed and used in small scale works
4. Temporary Bench Mark : reference points establish during levelling operation when there is a break in work

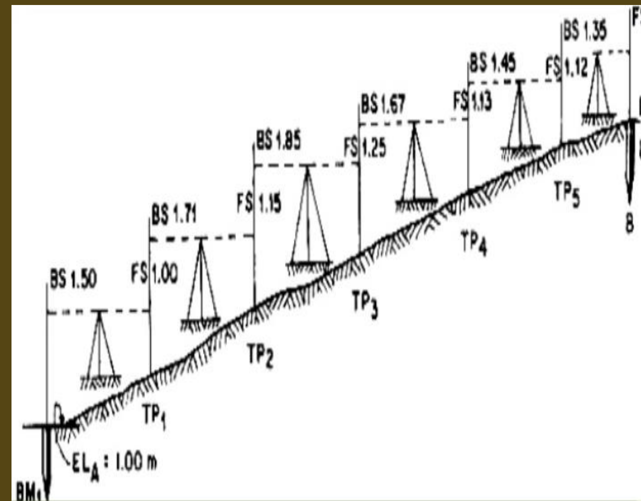


Classification of levelling

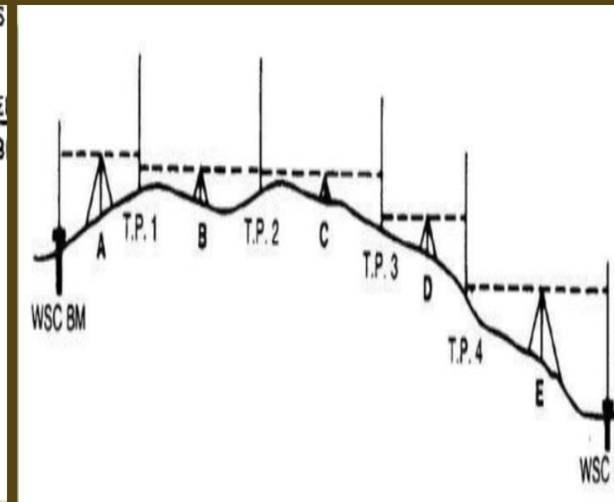
- Simple levelling : elevation between 2 points
- Differential levelling: elevation diff. when points are far apart
- Fly levelling : accuracy of levelling work (only BS & FS taken)
- Check levelling: accuracy of levelling work at the end of a day
- Profile levelling: levels along the centerline of any alignment
- Cross levelling: perpendicular to alignment at regular intervals
- Reciprocal levelling: points are far apart; unable to setup midway points
- Precise levelling: used for establishing BMs
- Trigonometric levelling: vertical distances found from horizontal distance and vertical angles
- Barometric levelling: altitude diff from pressure difference
- Hypersometric levelling: altitude diff. from temperature difference



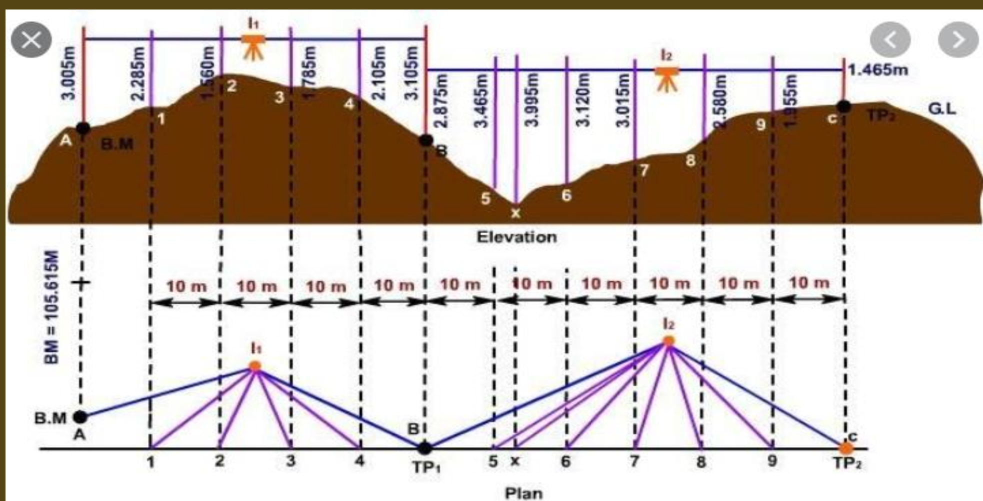
Datum
RL +50 m
2,554
1,786
0,927
1,983
3,589
1,305
1,422
3,250
0,571
1,925
3,015
0,462
0,780
A
B
C
D
E
F
G
H
J
Level position 1
Level position 2
Level position 3
Level position 4



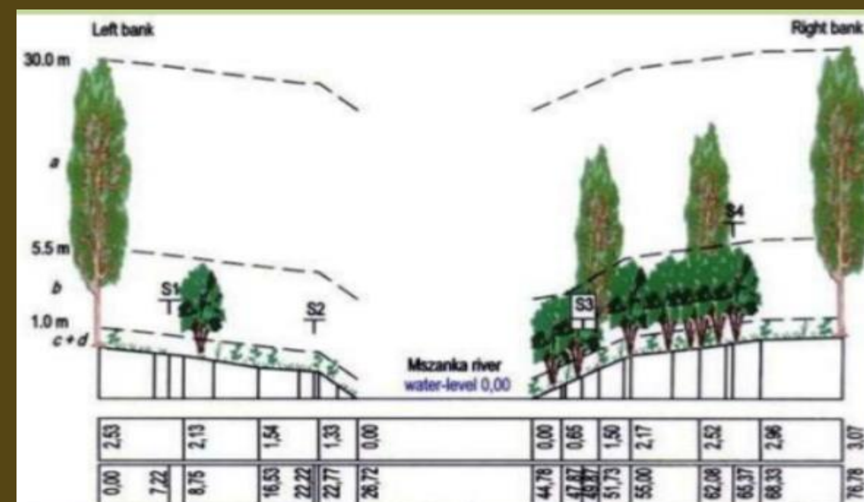
BS 1.50
FS 1.00
BS 1.71
FS 1.15
BS 1.85
FS 1.25
BS 1.67
FS 1.13
BS 1.45
FS 1.12
BS 1.35
FS
TP1
TP2
TP3
TP4
TP5
EL_A : 1.00 m
BM1



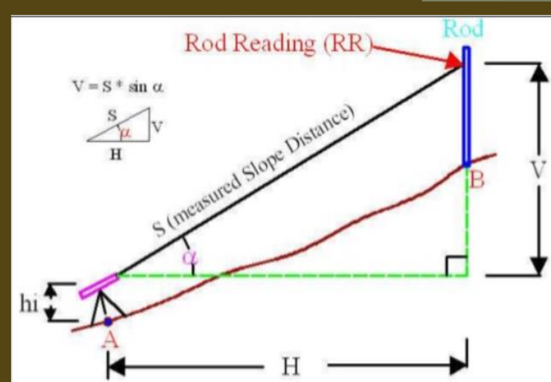
WSC BM
A
T.P. 1
B
T.P. 2
C
T.P. 3
D
T.P. 4
E
WSC BM



I1
I2
3.005m
2.285m
1.560m
1.785m
2.105m
3.105m
2.875m
3.465m
3.995m
3.120m
3.015m
2.580m
1.955m
1.465m
A
B.M
B
C
TP2
G.L
Elevation
BM = 105.615M
10 m
B.M
A
TP1
x
Plan



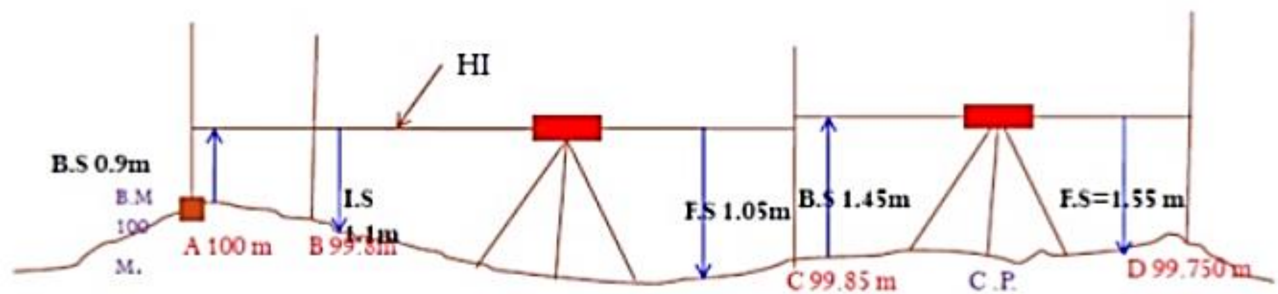
Left bank
Right bank
30.0 m
5.5 m
1.0 m
a
b
c+d
S1
S2
S3
S4
Mszanka river
water-level 0,00
2,53
2,13
1,54
1,33
0,00
0,00
0,65
1,50
2,17
2,52
2,96
3,07
0,00
7,22
8,75
16,53
22,22
22,77
26,72
44,78
51,73
55,00
62,08
65,37
68,33
76,78



Rod Reading (RR)
Rod
V = S * sin α
S
V
H
α
S (measured Slope Distance)
α
V
B
hi
A
H

Methods of reducing levels

1. Height of Instrument method
   - **This method consist of calculating HI for every instrument set up & then calculate the RL of point**
   - **H.I = R.L of B.M + BS**
   - **RL = H.I – IS (or FS)**
   - **Arithmetic check :** $\Sigma BS - \Sigma FS = last\ RL - first\ RL$



Station	B.S	I.S	F.S	H.I	R.L	Remark
A	0.9			100.9	100.00	B.M
B		1.1			99.800	
C	1.450		1.05	101.3	99.850	C.P.
D			1.550		99.750	

Q1) The following consecutive readings were taken with a level and a 4m staff on a continuously sloping ground at a common interval of 30 m.

0.680, 1.455, 1.855, 2.330, 2.855, 3.380, 1.055, 1.860, 2.265, 3.540, 0.835, 0.945, 1.530 and 2.250

Enter the readings as on a field book page, reduce the levels, apply checks and determine the gradient of the line. Use H.I method. RL of BM =80.750 m

Station	B.S	I.S	F.S	H.I	R.L	Remarks
1	0.680				80.750	BM
2		1.455				
3		1.855				
4		2.330				
5		2.885				
6	1.055		3.380			CP1
7		1.860				
8		2.265				
9	0.835		3.540			CP2
10		0.945				
11		1.530				
12			2.250			

For instrumental set up 1:

H.I = RL of BM + BS

= 80.750 +0.680 = 81.430 m

RL of station 2 = HI – IS

= 81.430 – 1.455

= 79.975 m

RL of station 6 = HI –FS

= 81.430 -3.380

= 78.050 m

For instrumental set up 2:

H.I = RL + BS

=78.050+1.055 = 79.105 m

Statn	B.S	I.S	F.S	H.I	R.L	Remarks
1	0.680			81.430	80.750	BM
2		1.455			79.975	
3		1.855			79.575	
4		2.330			79.100	
5		2.885			78.545	
6	1.055		3.380	79.105	78.050	CP1
7		1.860			77.245	
8		2.265			76.840	
9	0.835		3.540	76.400	75.565	CP2
10		0.945			75.455	
11		1.530			74.870	
12			2.250		74.150	
Sum	2.570		9.170			

Check: $\boldsymbol{\Sigma\, BS \;-\; \Sigma\, FS = last\ RL \;-\; first\ RL}$

Gradient = $\dfrac{Horizontal\ distance}{Difference\ in\ level} = \dfrac{11 \times 30}{6.6}$ = 50

Homework Question

The following consecutive readings were taken with a level and a 4m staff on a continuously sloping ground at a common interval of 20 m.

0.855 (on Q), 1.545, 2.335, 3.115, 3.825, 0.455, 1.380, 2.055, 2.855, 3.455, 0.585, 1.015, 1.850,1.850,2.755 and 3.845 (on R)

Enter the readings as on a field book page, reduce the levels, apply checks and determine the gradient of the line. Use H.I method

Q3) In running fly levels from a BM of RL = 250 m, the following readings were obtained:

B.S = 1.315; 2.035; 1.980; 2.625

F.S = 1.150; 3.450; 2.255

From the last instrument position, 5 pegs at 20 m intervals are to be set out on a uniform rising gradient of 1 in 40. The first peg RL is 247.245. Work out staff readings required for setting the top of the pegs on the given gradient.

Soln: Difference in level between consecutive stations

$$\frac{d}{r} = \frac{20}{40} = 0.5 \text{ m}$$

RL of 1st peg = 247.245 m

RL of 2nd peg = 247.245+0.5

= 247.745 m

RL of 3rd peg = 247.745+0.5

= 248.245 m

Statn	B.S	I.S	F.S	H.I	R.L	Remarks
1	1.315			251.315	250.000	BM
2	2.035		1.150	252.200	250.165	
3	1.980		3.450	250.730	248.750	
4	2.625		2.255	251.100	248.475	CP

Statn	B.S	I.S	F.S	H.I	R.L	Remarks
1	1.315			251.315	250.000	BM
2	2.035		1.150	252.200	250.165	
3	1.980		3.450	250.730	248.750	
4	2.625		2.255	251.100	248.475	CP
		X			247.245	1st peg
		X			247.745	2nd peg
		X			248.245	3rd peg
		X			248.745	4th peg
			X		249.245	5th peg

Staff readings:
For 1st peg, HI-X = RL
i.e., 251.100-X = 247.245
So, X = 3.855m

Statn	B.S	I.S	F.S	H.I	R.L	Remarks
1	1.315			251.315	250.000	BM
2	2.035		1.150	252.200	250.165	
3	1.980		3.450	250.730	248.750	
4	2.625		2.255	251.100	248.475	CP
		3.855			247.245	1st peg
		3.355			247.745	2nd peg
		2.855			248.245	3rd peg
		2.355			248.745	4th peg
			1.855		249.245	5th peg
Sum	7.955		8.710			

Q4) Determine the missing data

Statn	B.S	I.S	F.S	H.I	R.L	Remarks
1	?			201.740	200.000	BM at plinth level
2		?			199.245	Station A
3	2.140		3.180	?	198.560	CP
4		?			202.300	Inverted staff at lintel
5		2.020			?	Station B
6			?		?	BM at plinth level
Sum						

HI = RL of BM + BS

BS = HI – RL of BM = 201.740 – 200 =1.740 m

For station A,

RL = HI – IS

IS = HI –RL = 201.740 – 199.245 = 2.495 m

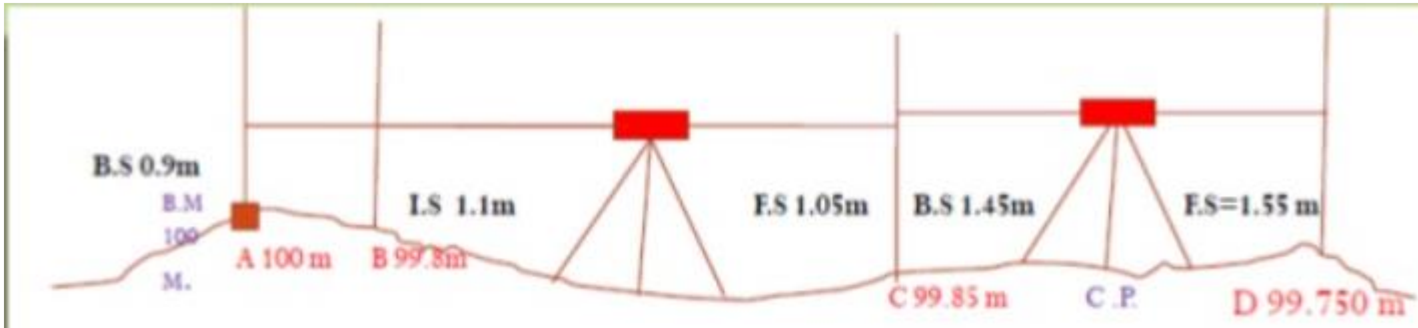
Statn	B.S	I.S	F.S	H.I	R.L	Remarks
1	?			201.740	200.000	BM at plinth level
2		?			199.245	Station A
3	2.140		3.180	?	198.560	CP
4		?			202.300	Inverted staff at lintel
5		2.020			?	Station B
6			?		?	BM at plinth
Sum						

Statn	B.S	I.S	F.S	H.I	R.L	Remarks
1	1.740			201.740	200.000	BM at plinth level
2		2.495			199.245	Station A
3	2.140		3.180	200.700	198.560	CP
4		-1.600			202.300	Inverted staff at lintel
5		2.020			198.680	Station B
6			0.700		200.000	BM at plinth
Sum	3.880		3.880			



Methods of reducing levels

2. Rise and Fall method
   - **Determining difference in level between consecutive points by comparing each point with immediate preceding point**
   - **RL = RL of preceding point + Rise (or - fall)**



Station	B.S	I.S	F.S	Rise	Fall	R.L	Remark
A	0.9					100.00	B.M
B		1.1			0.2	99.800	
C	1.450		1.05	0.05		99.850	C.P.
D			1.550		0.1	99.750	

Q5)The following consecutive readings were taken with a level and a 4m staff on a continuously sloping ground at a common interval of 20 m.

0.855 (on Q), 1.545, 2.335, 3.115, 3.825, 0.455, 1.380, 2.055, 2.855, 3.455, 0.585, 1.015, 1.850,1.850,2.755 and 3.845 (on R)

Enter the readings as on a field book page, reduce the levels, apply checks and determine the gradient of the line. Use Rise and fall method

Chainage	B.S	I.S	F.S	Rise	Fall	R.L	Remark
0	0.855					100	BM on Q
20		1.545					
40		2.335					
60		3.115					
80	0.455		3.825				
100		1.380					
120		2.055					
140		2.855					
160	0.585		3.455				
180		1.015					
200		1.850					
220		1.850					
240		2.755					
260			3.845				BM on R

Take difference between consecutive readings & if the value is positive enter in Rise or else in fall

For chainage 20m,

0.855 – 1.545 = -0.690

So, RL = 100 -0.690

= 99.310

For chainage 40m,

1.545 – 2.335 = -0.790

So, RL = 99.310-0.790

= 98.520 m

Chainage	B.S	I.S	F.S	Rise	Fall
0	0.855				
20		1.545			0.690
40		2.335			0.790
60		3.115			0.780
80	0.455		3.825		0.710
100		1.380			0.925
120		2.055			0.675
140		2.855			0.800
160	0.585		3.455		0.600
180		1.015			0.430
200		1.850			0.835
220		1.850			0.000
240		2.755			0.905
260			3.845		1.090

Chainage	B.S	I.S	F.S	Rise	Fall	R.L	Remark
0	0.855					100	BM on Q
20		1.545			0.690	99.310	
40		2.335			0.790	98.520	
60		3.115			0.780	97.740	
80	0.455		3.825		0.710	97.030	
100		1.380			0.925	96.105	
120		2.055			0.675	95.430	
140		2.855			0.800	94.630	
160	0.585		3.455		0.600	94.030	
180		1.015			0.430	93.600	
200		1.850			0.835	92.765	
220		1.850			0.000	92.765	
240		2.755			0.905	91.860	
260			3.845		1.090	90.770	BM on R
Sum	1.895		11.125	0.000	9.230		

Check: $\Sigma\, \boldsymbol{BS} - \Sigma\, \boldsymbol{FS} = \Sigma\, Rise - \Sigma Fall = \boldsymbol{last\ RL} - \boldsymbol{first\ RL}$

Gradient = 1 in 28.169

Q6) During fly levelling, the following readings were made:

BS = 0.62, 2.05, 1.42, 2.63 and 2.42 m

FS = 2.44, 1.35, 0.53 and 2.41 m

The first BS was taken on a BM of RL 100m. From the last BS, it is required to set 4 pegs each at a distance of 30 m on a rising gradient of 1 in 200. enter these notes on a level book and calculate the RLs of each peg top by rise & fall method. Also calculate the staff readings on the peg top

Station	B.S	I.S	F.S	Rise	Fall	R.L	Remark
1	0.62					100	BM
2	2.05		2.44				
3	1.42		1.35				
4	2.63		0.53				
5	2.42		2.41				
6							
7							
8							
9							

Statn	B.S	I.S	F.S	Rise	Fall	R.L	Remark
1	0.62					100	BM
2	2.05		2.44		1.82	98.18	
3	1.42		1.35	0.70		98.88	
4	2.63		0.53	0.89		99.77	
5	2.42		2.41	0.22		99.99	
6							
7							
8							
9							

Difference in level between two consecutive pegs,

$\frac{d}{r} = \frac{30}{200} = 0.15$ m

RL of 1st peg = 99.99 + 0.15 = 100.14 m

RL of 2nd peg = 100.14 + 0.15 = 100.29 m

And so on

Statn	B.S	I.S	F.S	Rise	Fall	R.L	Remark
1	0.62					100	BM
2	2.05		2.44		1.82	98.18	
3	1.42		1.35	0.70		98.88	
4	2.63		0.53	0.89		99.77	
5	2.42		2.41	0.22		99.99	
6		X		0.15		100.14	
7		X		0.15		100.29	
8		X		0.15		100.44	
9			X	0.15		100.59	

Staff reading for first peg :

2.42 – x = 0.15

Therefore x = 2.42 -0.15 = 2.27 m

And so on

Statn	B.S	I.S	F.S	Rise	Fall	R.L	Remark
1	0.62					100	BM
2	2.05		2.44		1.82	98.18	
3	1.42		1.35	0.70		98.88	
4	2.63		0.53	0.89		99.77	
5	2.42		2.41	0.22		99.99	
6		2.27		0.15		100.14	
7		2.12		0.15		100.29	
8		1.97		0.15		100.44	
9			1.82	0.15		100.59	
Sum	9.14		8.55	2.41	1.82		

Check: $\boldsymbol{\Sigma\, BS - \Sigma\, FS} = \Sigma\, Rise - \Sigma Fall = \boldsymbol{last\ RL - first\ RL}$

Q7) Fill in the missing data and perform the checks

Station	B.S	I.S	F.S	Rise	Fall	R.L	Remark
1	3.250					?	
2	1.880		?		0.600	?	
3		2.250			?	?	
4	?		1.920	?		?	
5		2.540			0.015	?	
6	?		?	1.000		?	
7	1.175		2.115		?	225.305	
8		1.625			?	?	
9	?		1.895		0.270	?	
10			1.255		0.750	?	
Sum	11.450		?	?	?	?	

3.250 - ? = -0.600

So, ? =3.250+0.600 = 3.850 m

Answer

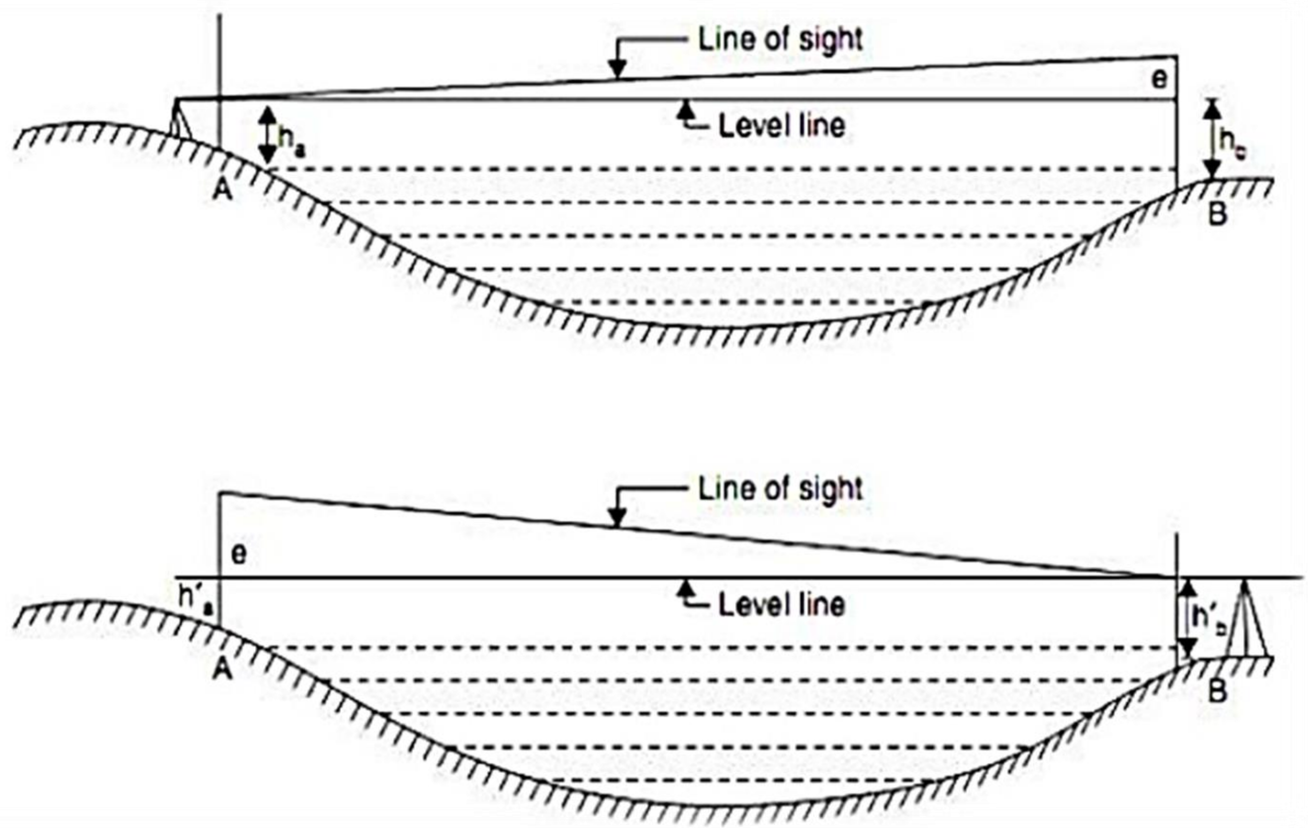
Station	B.S	I.S	F.S	Rise	Fall	R.L	Remark
1	3.250					?	
2	1.880		3.850		0.600	?	
3		2.250			0.370	?	
4	2.525		1.920	0.330		?	
5		2.540			0.015	?	
6	2.115		1.540	1.000		?	
7	1.175		2.115		0.000	225.305	
8		1.625			0.450	224.855	
9	0.505		1.895		0.270	224.585	
10			1.255		0.750	223.835	
Sum	11.450		12.575	1.330	2.455		

Answer

Station	B.S	I.S	F.S	Rise	Fall	R.L	Remark
1	3.250					224.960	
2	1.880		3.850		0.600	224.360	
3		2.250			0.370	223.990	
4	2.525		1.920	0.330		224.320	
5		2.540			0.015	224.305	
6	2.115		1.540	1.000		225.305	
7	1.175		2.115		0.000	225.305	
8		1.625			0.450	224.855	
9	0.505		1.895		0.270	224.585	
10			1.255		0.750	223.835	
Sum	11.450		12.575	1.330	2.455		

Reciprocal Levelling

- Used to obtain level difference between two points that are far apart
- Also used when it is not possible to set up level in midway between points





- Procedure
  - Let A & B be two points on opposite banks of a river whose level difference needs to be calculated
  - Set up level near A and take readings on A & B. Let this reading be a1 and b1
  - Shift the level to the other bank and set up the level near B and take readings on A &B. Let this reading be a2 and b2
  - Let h be the true difference in level between the points

$$\mathbf{h} = \frac{(\mathbf{b1} - \mathbf{a1}) + (\mathbf{b2} - \mathbf{a2})}{\mathbf{2}}$$

  - Combined error e is obtained as $\mathbf{e} = \frac{(\mathbf{b1} - \mathbf{a1}) - (\mathbf{b2} - \mathbf{a2})}{\mathbf{2}}$

Q8) In levelling across a river, two pegs A & B were fixed on opposite banks. The following readings were taken.

Inst. near station	Staff readings	
	A	**B**
A	1.871	1.469
B	1.664	0.706

If RL of A is 50.865, find the RL of point B

Solution:

True difference in level between the points

$$\mathbf{h} = \frac{(\mathbf{b1}-\mathbf{a1})+(\mathbf{b2}-\mathbf{a2})}{\mathbf{2}} = \frac{(1.871-1.469)+(1.664-0.706)}{2}$$

= 0.680 m (B being higher)

RL of B = RL of A + difference in elevation

=50.865 + 0.680

= 51.545 m

Q9) A dumpy level was set up with its eye piece vertically over a peg C. The height from the top of the peg C to the centre of eyepiece was measured and found to be 1.578 m. The reading on the staff held on peg D was 1.008 m. The level was then moved and setup likewise at peg D. The height of eyepiece above D was 1.258 m & the reading on the staff held on peg C was 1.812 m. Find the RL of D if that of C was 163.378.

Solution:

True difference in level between the points

$$\mathbf{h} = \frac{(\mathbf{b1}-\mathbf{a1})+(\mathbf{b2}-\mathbf{a2})}{\mathbf{2}} = \frac{(1.578-1.008)+(1.812-1.258)}{2}$$

= 0.562 m (D being higher)

RL of D = RL of C + difference in elevation

=163.378 + 0.562

= 163.940 m

Q8) Reciprocal levels were taken with a dumpy level & following observations were recorded:

Inst. near station	Staff readings	
	A	**B**
A	1.225	1.375
B	0.850	0.500

RL of A is known to be 626.155. Calculate RL of station B. Also calculate the error in line of collimation & state clearly whether it is inclined upwards or downwards

Solution

True difference in level between the points

$$\mathbf{h} = \frac{(\mathbf{b1}-\mathbf{a1})+(\mathbf{b2}-\mathbf{a2})}{\mathbf{2}} = \frac{(1.375-1.225)+(0.500-0.850)}{2}$$

= 0.100 m (A being higher)

RL of B = RL of A - difference in elevation

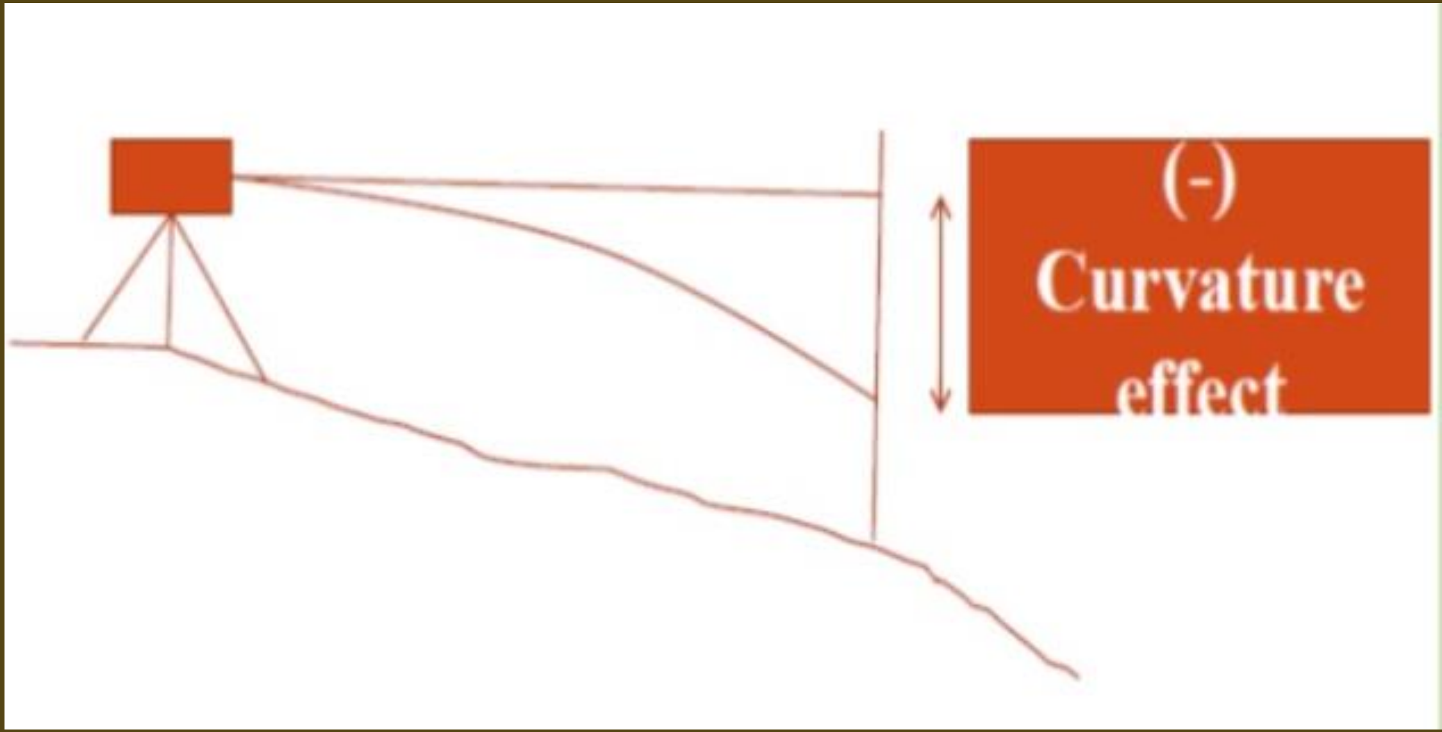
=626.155 - 0.100

= 626.055 m

Corrections due to curvature & refraction

- Correction due to curvature

- For long sights the curvature of earth can effect staff readings. The line of sight is horizontal but the level line is curved and parallel to the mean spheroidal surface of the earth.
- The vertical distance between the line of sight and level line at particular place is called the curvature correction
- The effect of curvature is to cause the object sighted to appear lower than they really are.
- Curvature correction is always **Subtractive(-)**
- **True staff reading=(Observed staff reading-0.0785D^2)m**
- Where D= distance in Km.


(-)
Curvature
effect

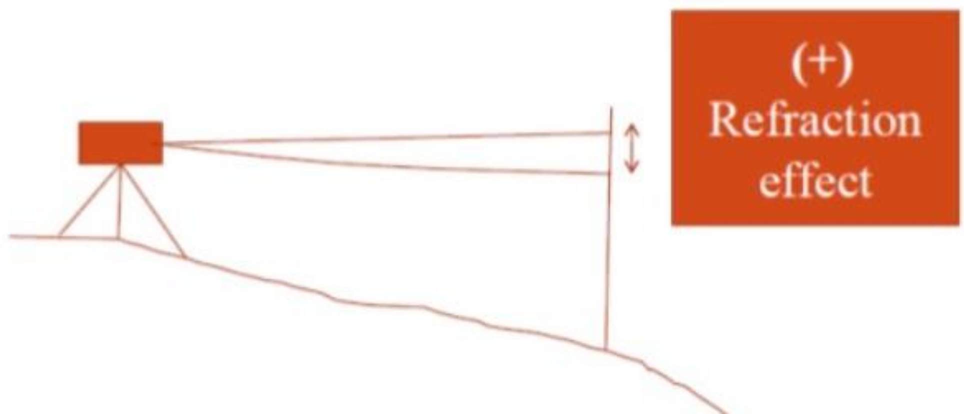
➢ Refraction correction

- The ray of light pass through layers of air of different densities and refractor bent down. The effect of refraction is to make the object appear higher then they really are. Refraction varies considerably with climate conditions.

However it is taken as,

- **$C_r = 0.0112\ D^2 m (+)$**
- Refraction is always additive
- **True staff reading**
- **=Observed staff Reading+ Refraction correction.**

Combined Correction = $0.0673\ D^2$





Q) A level was set up at a point O and the distances to 2 staff stations A & B were 150 m & 250 m respectively. The observed staff readings on stations A & B were 2.725 and 1.855. Find the correct differences of levels between stations A & B.

Combined Correction for staff reading A= $0.0673\ D^2$

$= 0.0673 \times (150/1000)^2 = 0.0015$ m

Similarly for staff reading B = 0.0042 m

Correct staff reading on A = 2.7250 – 0.0015 = 2.7235 m

Correct staff reading on B = 1.8550 – 0.0042 = 1.8508 m

Correct level difference between A & B = 2.7235 -1.8508

= 0.8727 m



Distance to visible horizon

- Let C be the staff station; BC = h be the staff reading; d be the distance to visible horizon
- Let the horizon meets the earth surface at A, the point where the level line CA meets the horizon of point B

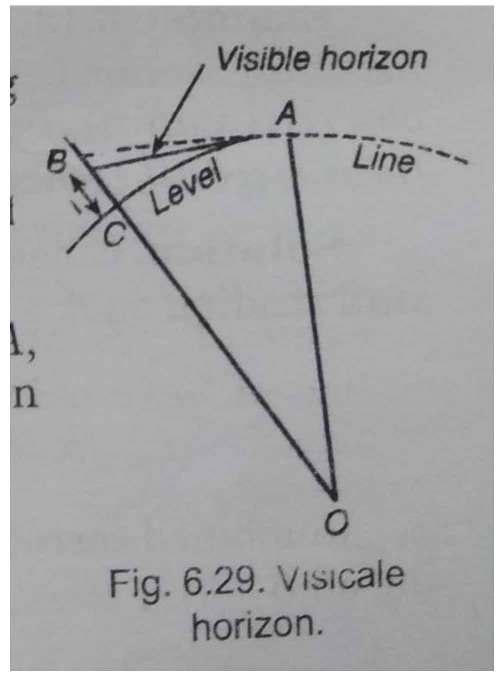


Fig. 6.29. Visicale horizon.

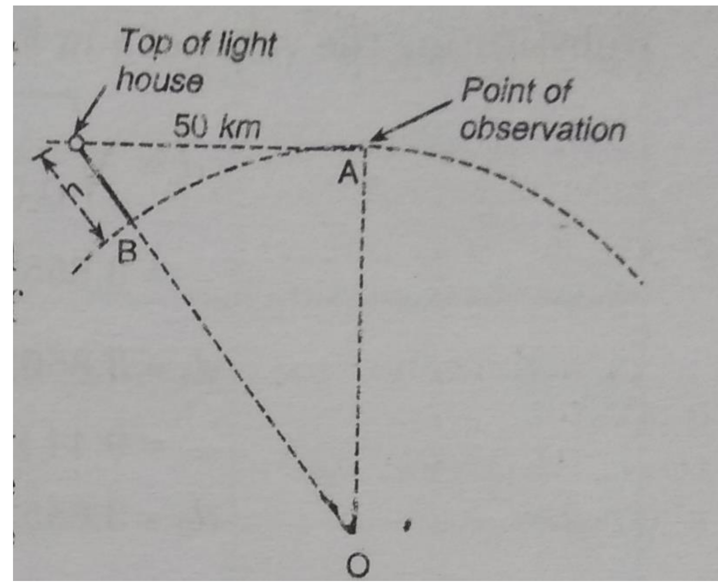
$$d = 3.8553\sqrt{h} \text{ km}$$

Q) A lamp on the top of a light house is visible just above the horizon at a certain station at sea level. The distance of the top of the light house from the station of observation is 50 km. Find the height of the lamp above sea level.

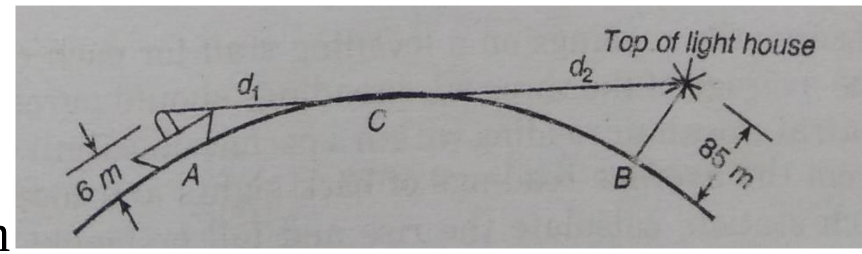
d = $3.855\sqrt{h}$ km

So, h = $0.0673\ D^2$

$= 0.0673 \times 50^2 = 168.25$ m



Q) From the deck of a ship, the light at the top of a light house is visible just above the horizon. The heights of the top of the light house and the eye of the viewer from the ship above mean sea level may be assumed as 85 m and 6 m respectively. Assuming the radius of earth as 6370 km and the usual correction refraction, determine the distance between the ship and light house.



d = 3.8553$\sqrt{h}$ km

d1 = 3.8553 $\sqrt{6}$ = 9.44 km

d2 = 3.8553 $\sqrt{85} = 35.54\ km$

Total distance AB = d1 + d2 = 44.98 km