

## Meaning and Nature of Mathematics

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### Meaning

- Greek word – *máthēma* – *learning, study, science*, and additionally came to have the narrower and more technical meaning "mathematical study".
- Ancient Hindu - "Ganita" – "science of calculations"
- To a philosopher – "Mathematics is all types of deductive reasoning"
- It deals with quantitative facts, relationships as well as with problems involving space and form.
- It is a logical study of shape, arrangement and quantity.

### Definition

- **According to Locke –**  
"Mathematics is a way to settle in mind a habit of reasoning"
- **According to Webster's New World Dictionary (1973) –**  
"Mathematics is the science dealing with quantities, forms etc and their relationships by the use of numbers and symbols."
- **According to Goods Dictionary of Education (1984) -**  
"Mathematics is the science of measurement, quantity and magnitude."
- **According to Chamber's Twentieth Century Dictionary (1987) -**  
"Mathematics is the science of Magnitude and number and of all their relations."
- **According to Bhargawas Standard Illustrated Dictionary (1990)**  
"Mathematics is the science of space and number."

### Analysis of Definition

- science of number,
- structure (sets of numbers, functions)
- space (geometry)
- Measurement,
- quantity (numbers- natural, whole, integers, rational, real etc.)
- magnitude.

*It is systematized, organized and exact branch of science.*

- Utilitarian View-tool for solving problem
- Purist View-Study of pattern, relation and interconnected ideas
- Language, tool for a set of standards
- More than subject, prerequisite for everything – values → disciplinary, practical, cultural, social, moral
- Provides ways to model real situation → solar eclipse, pyramids, globe (lati and longi), model of eye, ear, brain, planets, working models
- Scientific and technological
- Common view- to communicate
- Language to think in a different way
- Mathematics statements are unambiguous
- Mathematics is consistent
- Conceptual and procedural
- Important life skill
- Continue learning

### Characteristics of Mathematics

- Mathematics is an abstract science.

- It is a tool for dealing with abstract concept of any kind
- Science of number and space.
- It deals with quantitative facts and relationships.
- It deals with problems involving space and form.
- It establishes various relationships between phenomenon in space.
- It has its own language – signs, symbols, terms & operation and hence can handle ideas with precision and conciseness that is unknown to any other language.
- It helps to give exact interpretation to his ideas and conclusions.
- It explains that science is a by-product of our empirical knowledge.
- It involves man's high cognitive powers.
- It has its own tools like intuition, logical reasoning, analysis, construction, generalities and individuality.
- It is a science of logical reasoning.
- It helps in drawing conclusion & interpreting various ideas & themes
- It helps in solving the problem of our life

**Mathematics is not only 'number work' or 'computation', but is more about forming generalizations, seeing relationships and developing logical thinking and reasoning.**

## **Nature of Mathematics**

### **Science of Logical Reasoning**

- Logic is an important factor in mathematics
- According to Russel and Whitehead **mathematics is logic**
- Mathematics has few premises on which we base our reasoning.
- It governs the pattern of deductive proof through which mathematics is developed.
- All proposition which mathematicians enunciate can be deduced one from the other by the rules of formal logic.
- Proceeds from simple to complex, concrete to abstract
- Dependence on earlier knowledge is particularly great
- Arithmetic  $\square$  Algebra  $\leftarrow$  Calculus  $\leftarrow$  Dynamics .....
- Graduation and sequence observed among topics in any branch of mathematics
- Helps to prove results in geometry
- Types of proofs
  - **Direct proof:** it proceeds from propositions already accepted
  - **Indirect proof:** if **p** is to be proved, assume **not p** is true and hence derive a contradiction. Like  $\sqrt{2}$  is irrational
  - **Proof using contra-positive:** "If two lines cut by a transversal are not parallel, then the interior alternate angles are not equal"
  - **Disproof by counter:** simply exhibit a counter example. E.g. the sum of two odd numbers is odd.

### **Induction or Inductive reasoning**

- Outcomes of observations of mathematical phenomenon, relationships and experiences is Generalization.
- **Principle:** if a relationship holds good for some particular cases, it holds good for any similar case and hence the relationship can be generalized. Process is called inductive reasoning.
- Many mathematical rules, formulae, definitions can be generalized through inductive reasoning.
- Number of Examples  $\rightarrow$  observe relationship  $\square$  generalization

### **Deduction or Deductive reasoning**

- Mathematics is a system of logical processes whereby conclusions are deduced from certain fundamental assumptions and definitions that have been accepted or proved to be true.
- The student draws inferences from the premises
- Example: "When two lines intersect, vertically opposite are equal"

## **Essentials of Deductive systems**

- **Undefined terms:** terms which defy definitions e.g. point, straight line
- **Definitions:** some technical terms, definitions have to be framed e.g. definition of an angle, triangle, tangent
- **Postulates:** First principles which are taken for granted and have no self evident truth. General truth, common to all studies .
  - Example: A straight line may be drawn from any point to any other point
  - Two straight lines cannot intersect at more than one point.
- **Axioms:** An **axiom** is a premise or starting point of reasoning. As classically conceived, an axiom is a premise so evident as to be accepted as true without controversy.
  - **Axioms:** truth related to special study at hand and initial assumptions.
    - Example: if  $a > b$ ,  $b > c$  then  $a > c$
    - Things equal to the same thing are equal to one another

## **Difference between an Axiom and Postulate**

- An axiom is in some sense thought to be strongly self-evident.
- A "postulate," on the other hand, is simply postulated, e.g. "let" this be true. There need not even be a claim to truth, just the notion that we are going to do it this way and see what happens.

## **Language and symbolism**

- Mathematics has its own language and symbols.
- Language for communicating ideas is largely in terms of symbols and words which everybody cannot understand.
- Mathematical talks consists of making use of mathematics symbolism.
- Acquiring the language is essential for children.
- Long period of training and endurance.
- Makes mathematical language more elegant and precise than any other language.
- Example: commutative law of addition and multiplication: 'the addition and multiplication of two real numbers is independent of the order in which they are combined' ----- *Concise form ???*
- All mathematical statements, relations and operation – mathematical symbols. *Add an example*

Any one, who wants to read and communicate effectively in mathematical language, has to be well versed in the mathematical symbols and their definite uses.

## **Mathematics is a science of Discovery**

- **E.E. Biggs, 1963:** Mathematics is the discovery of relationships and expression of those relationships in symbolic form- in other words, in numbers, in letters, by diagrams or by graphs
- **A. N. Whitehead:** "Every child should experience the joy of discovery".
- Mathematics gives an easy and early opportunity to make independent discoveries.
- Children should get opportunities for making discoveries as well as practice to achieve accuracy.
- Discovery technique is making spectacular progress.
- Application in pure mathematical relationships and everyday problems

## **Mathematics as an Abstract science**

- Mathematical concepts are abstract – they cannot be seen or felt in the physical world.
  - E.g. Euclid's lines are supposed to have no width, his points no sides → no such object found
  - Infinity is never experienced; yet it is the central concept of mathematics
  - Negative numbers do not correspond any physical objects because we cannot have quantities less than nothing

## **Mathematics as an Abstract science**

- All mathematical concepts cannot be learned through experiences with concrete objects. Some can be learned only through their definitions and may not have concrete counterparts to be abstracted from.
- Most of the mathematical concepts are without concretization; hence abstract
  - E.g. Concept of prime numbers, probability, function, limits, continuous function

- Even when concretized; just the representation of concepts and not physical objects themselves

### **Intuition**

- Intuition involves concretization of idea not yet stated in the form of operations or example.
- Anticipate what will happen next and what to do about it.
- Development of intuition
- Applicability of the process rather than the importance of right answers all the time.

### **Science of precision & accuracy**

- Mathematics of known as an exact science because of its precision.
- It is perhaps the only subject which can claim certainty of results.
- In mathematics the results are either right or wrong, accepted or rejected.
- There is no midway possible between right and wrong.
- Mathematics can decide whether or not its conclusions are right.
- Even when there is new emphasis on approximation, mathematical results can have any degree of accuracy required.
- It is the teacher's job to help the students taking decision.

### **Deals with Generalization & classification**

- Mathematics provides ample exercises in combining various results under one head, in making schematic arrangements and classification
- When the pupils evolves his own definitions, concepts and theorems, he is making generalization
- The generalization and classification of mathematics is very simple and obvious in comparison with those of other domains of thought and activity.
- However the mathematics teacher should take care to see that the final generalization into a rule should always be deferred until it is almost spontaneously suggested by the pupil themselves

### **Mathematical Statements are unambiguous**

- All objects, concepts, statements have clear and precise meaning.
- There is no space for vagueness.
  - Sphere deals with roundness and the sense of roundness of a sphere is different from the sense of roundness in circle.
  - We know very well that we will not give a bangle as an example of sphere and a ball as an example of circle.
- In mathematics all statements, be a definition or a sentence stating a result, are unambiguous
- Mathematics is a human Endeavour.
- Humans have formally constructed the abstract world of mathematics resolving ambiguity at every point by carefully choosing axioms, constructing definitions of mathematical objects that define concepts, formulating statements that tell us about interrelations and interconnections between different mathematical objects and mathematical statements.
- We can always tell about mathematical statements whether they are true or false.
- This gives power to mathematical knowledge as it does not rest on individual opinions.

### **Mathematics is Consistent**

- In mathematics, truth is a matter of consistency which is seen through use of logic.
  - E.g. sum of two odd numbers is an even number
- Truth of a statement in mathematics depends on the accepted truths and is accepted on the basis of logic.

### **References**

- **Reference books-** *Teaching of Mathematics- S. K. Mangal, Anice James, Sonia Bhasin*

## Aims and Objectives of teaching mathematics

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- o The **National Curriculum Framework (NCF 2005)** is one of four National Curriculum Frameworks published in 1975, 1988, 2000 and 2005 by the National Council of Educational Research and Training (NCERT) in India.
- o The document provides the framework for making syllabii, textbooks and teaching practices within the school education programmes in India.
- o The approach and recommendations of NCF-2005 are for the entire educational system.

### According to NPE – 1986 (modified in 1992)

“Mathematics should be visual as the vehicle to train a child to think, reason, analyse and to articulate logically. Apart from being a specific subject, it should be treated as a concomitant to any subject involving analysis and reasoning. With the recent introduction of computers in schools, educational computing and the emergence of learning through the understanding of cause-effect relationships and the interplay of variables, the teaching of mathematics will be suitably redesigned to bring it in line with modern technological devices.”

### Objectives

- o **What? →** Objectives are definite, functional and workable units. Short-term, immediate goals and purposes that may be achieved within the specified classroom situation.
- o **Why →** Help in the ultimate realization of *Aims of teaching mathematics* (aims are ideals, need long term planning, realization is not an easy task)
- o The **National Curriculum Framework (NCF 2005)** is one of four National Curriculum Frameworks published in 1975, 1988, 2000 and 2005 by the National Council of Educational Research and Training NCERT in India.
- o The document provides the framework for making syllabii, textbooks and teaching practices within the school education programmes in India.
- o The approach and recommendations of NCF-2005 are for the entire educational system.
- o Compulsory subject from class I to Secondary level.
- o Has significance due to its relation to several vocations, its correlation to other subjects and life.
  - o As per NCF-2005, the main goal of math education is the development of children's ability of mathematisation.
  - o i.e. Children should learn to think about any situation using the language of mathematics so that the tools and techniques of mathematics can be used. This typically involves drawing pictures (representations), choosing variables, framing equations and arriving at a conclusion logically.

### Aims of teaching mathematics

- o The narrow aim of school mathematics is to develop 'useful' capabilities, particularly those numeracy-related skills such as the ability to deal with arithmetic operations, ability to compute percentage, area, volume, to factor polynomials etc., and the mathematics required to handle abstraction.
- o Skills needed to transact one's daily life business along with social obligations smoothly. It is of immediate need
- o The higher aim is to develop the
  - o child's capability for logical and analytical thinking, nurturing a confident attitude to problem solving, and an ability to decide which mathematical tools are appropriate in which context and to apply them accordingly.
  - o to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction.
- o It includes a way of doing things, and the ability and the attitude to formulate and solve problems.
- o These are important to deal with in the modern complex technological world.
- o **NCF-2005 talks about teaching ambitious, coherent and important mathematics .**

- An ambitious mathematics seeks to achieve the higher aim rather than only the narrower aim.
- Coherent means linkages of mathematics with other subjects.
- Teaching important mathematics means that it is not merely textbook material but something both children and teachers consider worth spending their time and energy on, and mathematicians consider significant for mathematics.
- An important consequence of such requirements is that school mathematics must be activity oriented.

### **Objectives for Secondary Level**

- To create a taste for mathematics so that learning mathematics becomes enjoyable and hence can be used and enjoyed lifelong.
- To develop an understanding of using various mathematical techniques for solving problems
- To make mathematics a part of children's life experiences. Something to talk about, to communicate, to discuss among themselves, to work together on.
- To develop the skill of problem solving. Not only solving meaningful problems but to make interesting problems too. Ability of use in one's life.
- To treat a child like a mathematician and allow him to construct his own knowledge.
- To develop logical thinking among children by inculcating habit of thought and communication in them.
- To help the children to understand the basic structure of mathematics. Algebra, Geometry, Arithmetic offers a methodology of abstraction, structuration and generalization. It will help in appreciating the scope and power of mathematics thereby refining our instincts.
- To help the teachers to engage the children in classroom activities. Adequately challenging the talented as well as participation of other students. Offer teachers the means and resources to organize learning experiences.

### **Objectives for Higher Secondary Level**

- To provide students with an appreciation of the wide variety of the application of Mathematics, and equip them with the basic tools that enable such application.
- To develop ability in students to decide which mathematical tools are appropriate in which context and to apply them accordingly.
- To nurture a confident attitude of problem solving
- To develop the child's capability for reasoning, logical and analytical thinking.
- To help the children to construct new knowledge from experience & prior knowledge using conceptual element. In order to make students active learners.

### **Reference**

- The teaching of mathematics; Kulbir Singh Sidhu
- [http://en.wikipedia.org/wiki/National\\_Curriculum\\_Framework\\_%28NCF\\_2005%29#cite\\_note-1](http://en.wikipedia.org/wiki/National_Curriculum_Framework_%28NCF_2005%29#cite_note-1)
- <http://www.ncert.nic.in/rightside/links/pdf/framework/english/nf2005.pdf>

## VALUES OF TEACHING MATHEMATICS

### UNIT 2C

Mathematics is a science of number and space. Mathematics has its own language, its own tools and mode of operation.

#### **Values / Importance of mathematics:**

##### **Utilitarian or Practical value:**

Not a single aspect of our life is free from its use. Its importance is not to that of mother tongue. People can go along without their mother tongue but not without calculation. The need of mathematics is felt right from the early morning by everyone of us. One has to get up early at the right time. There are so many engagements for the day that are to be carried out with precision in time. Mathematics is needed by all of us in every sphere of one's life. Mathematics, in one sense, has bread and butter value. One has to choose and train oneself in one or the other vocation. With the advent of modern technology, mathematics has been a living wire for one's contact to an occupation. The base of any occupation is economics and mathematics.

Mathematics is the gate and key of sciences. Today what we enjoy in our life as a result of scientific inventions has only been possible through mathematics. Moreover, mathematical knowledge is indispensable in understanding and controlling the forces of nature. Even the stars, sun, planets and the moon are bound to move as mathematics directs.

##### **Social value:**

Mathematics is a subject of great social importance. It helps in the proper organisation and maintenance of our social structure. Mathematics helps not only in the formation of laws but also in their compliance. The world transaction, exchange, commercial trade and business depend on Mathematics. One can lead a normal social life only when he is able to adjust himself in the existing social set-up. Today our social set-up is totally governed by the scientific and technological knowledge which can only be attained by the study of mathematics. Napoleon has rightly said "The progress and the improvement of mathematics are linked with the prosperity of the state".

##### **Moral Value:**

The qualities like truthfulness, honesty, purity of thoughts and cleanliness, justice, punctuality, patience, self-control, self-respect, self-confidence, respect for others opinion, discrimination between good and bad, observation of rules and belief in the systematic

organisation and arrangement etc. which definitely contribute towards a strong character are inculcated through the teaching of Mathematics. Mathematics helps in developing proper moral attitudes as there is no place of prejudiced feelings, biased attitude, doubts and half truth in the solutions of problems in mathematics.

Mathematics brings changes in a student that he becomes almost free from the vices of jealousy, envy and hatred.

### **Aesthetic value:**

Mathematics have developed a wrong notion in the minds that mathematics is a dry and uninteresting subject. For the lover of mathematics, there is all beauty, art, music and fineness in this subject. One finds a huge treasure of pleasure after getting success in the solution of Mathematics problem. There is no exaggeration in saying that mathematics is the creator as well as the nourisher and saviour of all the arts. What we enjoy in the arts like drawing, painting, music, or dance is all due to mathematics. Mathematical regularity, symmetry, order and arrangement play a leading part in beautifying and organising the work of these arts. Music is nothing but the mathematically organised sound. All the musical instruments are played on the set rules of mathematics. In dancing too one has to take care of mathematics in taking steps and responding to the tunes.

### **Intellectual value:**

The study of mathematics helps us to develop all our intellectual powers. There is no other subject in the curriculum like mathematics which makes student's brain active. Problem solving helps us in development of mental faculties. Mental work is needed for solving mathematical problem. Each problem of mathematics possesses such a sequence which is necessary for constructive and creative process. In this way all mental abilities of child are developed through mathematics. It provides all concentration, originality, power of discovery, thinking power, reasoning power, self-reliance and hard work etc.

### **Disciplinary value:**

Mathematics is not meant only for development of mental abilities but also to develop their personality with some qualities like concentration, truthfulness, seriousness etc. A person who is gaining mathematical knowledge is not in favour of working against the rules of under sentimental situations. A child judges about his good or bad with the help of his reasoning power, wisdom, patience, and self-confidence. Mathematics is the only subject whose knowledge develops the habit of hard work, concentration, well organised and clarity in the students. These are such conditions which enable the students of mathematics to lead seriously, wisdom, and disciplined life. It develops thinking and reasoning power and demands less from memory.



### **Cultural value:**

Each nation or society reflects its culture by its living standards rituals, artistic progress economic, social, and political aspects etc. The history of mathematics presents the image of culture of different nations. The person is said to be cultured if one is well educated and have refined manners of dealing. The person becomes critical observer, logical thinker and proper knowledge of mathematics changes the mind of the person. Thus the person becomes more cultured with the proper knowledge of mathematics. Mathematics not only familiarises us with culture and civilization but also helps in preserving, promoting cultural heritage and transmitting it to future generations. Through the application of scientific and mathematical discoveries our culture and civilization is undergoing constant change. The welfare of our civilization is now almost wholly dependent upon scientific as well as mathematical progress. It affects view of life and way of living as a result of which it also affects our philosophy of life. Hence the teaching of mathematics plays a vital role in developing our cultural heritage.

### **Vocational value:**

The main aim of education is to help the children to earn their living and to make them self-dependent. To achieve such aim, mathematics is the most important subject than any other subject. At present the vocational value of engineering, technology, management, information technology has become more important and prestigious or reputed. The knowledge and training of these vocations is possible only through mathematics. Almost each and every vocation needs the knowledge of mathematics. Eg to become an engineer, accountant, banker etc there is need of mathematical knowledge.

### **Psychological value:**

Mathematics education is also useful from the point of view of psychological aspects. Mathematics fulfils the psychological needs of the children. In mathematics emphasis is given on operations and drill work so that its knowledge becomes more solid as well as durable. The teaching of mathematics follows the various laws and principles of psychology such as learning by doing, learning through experiences and problem-solving etc. Through its knowledge the child develops and satisfies his desires, creative and constructive tendencies, self-satisfaction, self-assertion etc.

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