AREAS & VOLUMES

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Definition

- Area – area of a tract of land projected upon a horizontal plane and not in the actual area of the surface of land
- Units

  - Square metre : area of a square whose each side is 1 m
  - Are : area of a square whose each side is 10 m
  - Hectare : area of a land containing 100 square ares or 10000 m2
  - Square foot: area of a square whose each side is 1 foot
  - acre: area of land containing 4840 sq. yards or 43560 sq. feet

Determination of Area

- Areas can be calculated from

i) field notes

a. from regular geometric shapes

b. Area between the survey lines and irregular boundaries can be calculated from a number of offsets taken to the boundary

ii)from plotted plan /map

a. Graphical method

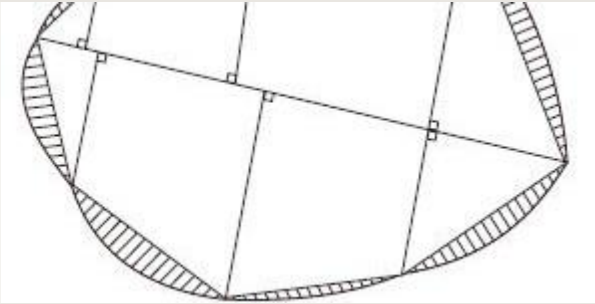
b. Instrumental method

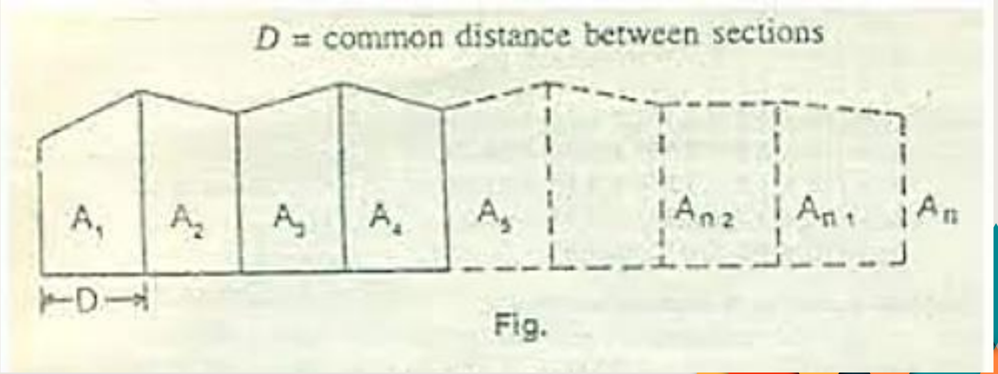
Area between a straight line & irregular boundary

<u>2 STEP PROCEDURE</u>

- Divide the base line into a number of equal parts
- Draw ordinates at each points of division and scale off their lengths (if necessary)

Required area can be calculated using any one of the four rules





Rules for area calculation

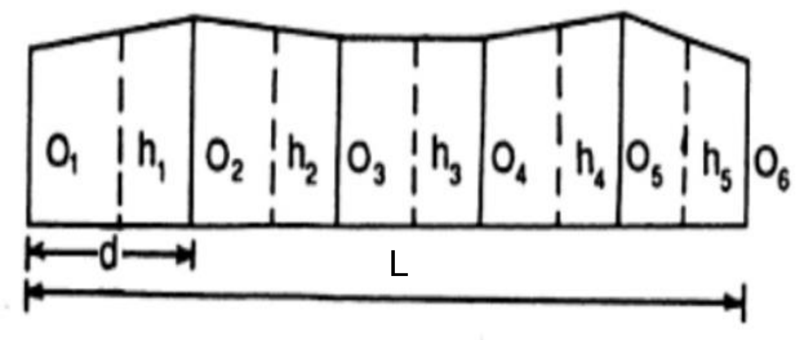
1. Mid Ordinate rule

If L = scaled length of base line AB

n = no. of divisions of base line

d = common distance between ordinates

$h_1, h_2 \ldots\ldots h_n$ = mid ordinates

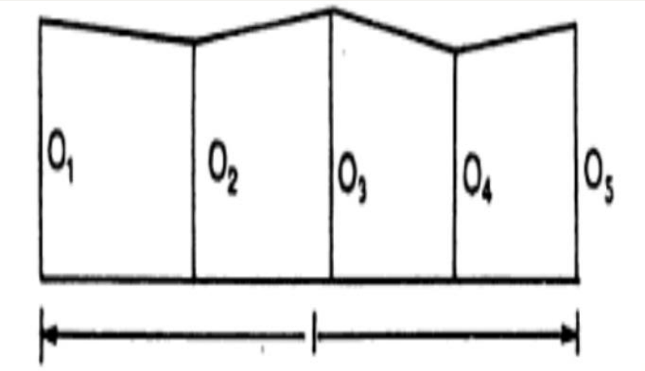


Area of the plot = $(h_1 + h_2 + h_3 + \ldots + h_n)$d

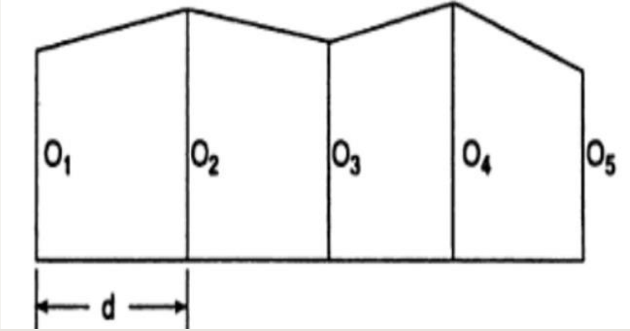
Rules for area calculation

2. Average Ordinate rule

- Let, O_1, O_2, O_3..... O_n= Ordinates Regular Intervals
- l= length of base line,
- n= number of divisions,
- n+1= number of ordinates
- Area= $\dfrac{O_1 + O_2 + O_3 \ldots\ldots O_n}{O_{n+1}} \times l$
- **Area= $\dfrac{\text{Sum of Ordinates}}{\text{No of Ordinates}}$ x length of base line**



Rules for area calculation



3. Trapezoidal rule

- Assumption: boundary between ends assumed to be straight and area enclosed between survey line and boundary line are considered as trapezoids

Let, $O_1, O_2, O_3 \ldots O_n$ = Ordinate at equal intervals

d = common distance

1^{st} Area= $\dfrac{O_1 + O_2}{2} \times d$

2^{nd} Area= $\dfrac{O_2 + O_3}{2} \times d$

3^{rd} Area= $\dfrac{O_3 + O_4}{2} \times d$

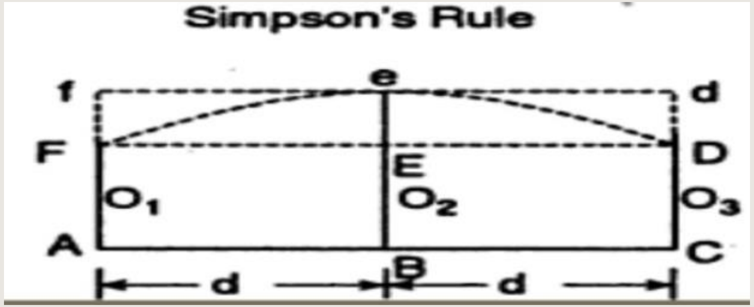
Last Area= $\dfrac{O_{n-1} + O_n}{2} \times d$

Total Area = $\dfrac{d}{2} [O_1 + 2O_1 + 2O_2 \ldots 2O_{n-1} + O_n]$

= $\dfrac{\text{Common distance}}{2}$ (1^{st} ordinate + Last Ordinate+ 2 (sum of other Ordinate)

" To the sum of 1^{st} & last ordinates, add twice the sum of remaining ordinates. Multiply the total sum by the common distance between the ordinates. Half of this product equals the required area"

Rules for area calculation



4. Simpson's rule (Parabolic rule)

- Assumption: portion of irregular boundary between ordinated are assumed to be arcs of parabola
- Applicable only when the number of ordinates is odd

Total Area= $d\,(O_1 + O_n + 4\,(O_2 + O_4 + \ldots + 2\,(O_3 + O_5 + \ldots)$

$= \frac{\text{Common distance}}{3}$ (1st ordinate + last Ordinate)
+ 4 (sum of even ordinates)
+ 2 (sum of remaining odd Ordinates)

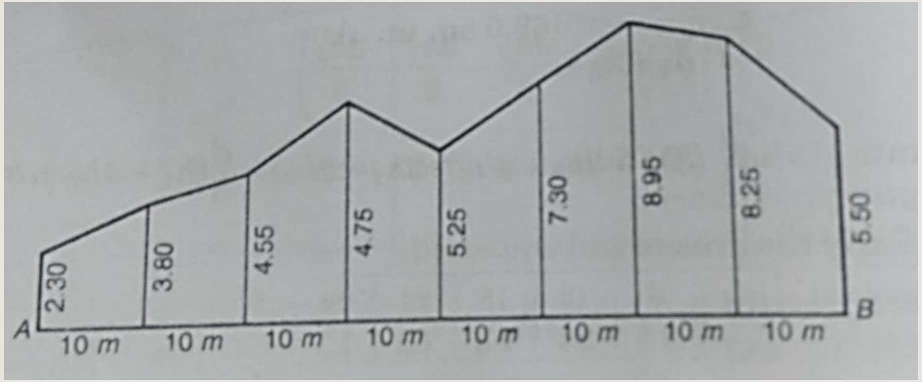
" To the sum of 1st & last ordinates, add twice the sum of remaining odd ordinates and four times the sum of the even ordinates. Multiply the total sum by one third the common distance between the ordinates. The result gives the required area"

Q1) The following perpendicular offsets were taken at 10 m intervals from a survey line AB to an irregular boundary line:

2.30, 3.80, 4.55, 6.75, 5.25, 7.30, 8.95, 8.25 and 5.50 m

Calculate the area enclosed by the application of

(i) Avg. ordinate rule, (ii) Trapezoidal rule, (iii) Simpson’s rule



Q2) A series of offsets were taken from a chain line to a curved boundary line at an interval of 10 m in the following order:
0, 2.85, 3.95, 6.45, 8.60, 8.90, 5.25, 0 m
Calculate the area between the chain line and curved boundary line by the simpson's rule

Step 1: Find area considering the offsets, O1 to O7, using Simpson's rule

Area – 515.75 sq. m

Step 2: Considering last 2 ordinates, calculate the area of the enclosed trapezium

Area – (O7+O8) x d/2 = 26.25 sq. m

Step 3: Total area

Area of plot = area using Simpson's rule + 26.25 = 542 sq.m

Q3) The following offset were taken from a chain line to an irregular boundary line at an interval of 10 m

0, 2.5, 3.5, 5.0, 4.6, 3.3, 0 m

Compute the area using (i) mid – ordinate rule (ii) average ordinate rule (iii) trapezoidal rule (iv) Simpson's rule

By mid-ordinate rule: The mid- Ordinate are

$$H_1 = \frac{0+2.5}{2} = 1.25.$$

$$H_2 = \frac{2.5+3.5}{2} = 3.00$$

$$H_3 = \frac{3.5+5.00}{2} = 4.25 \text{ m}$$

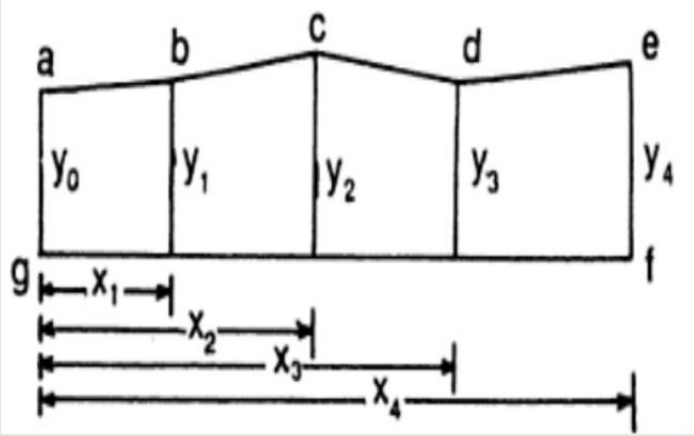
$$H_4 = \frac{5.00+4.6}{2} = 4.8 \text{ m}$$

$$H_5 = \frac{4.6+3.2}{2} = 3.9 \text{ m}$$

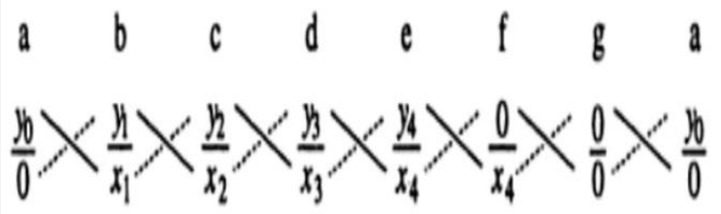
$$H_6 = \frac{3.2+0}{2} = 1.6 \text{ m}$$

Calculation of areas from co-ordinates

- Applied when Trapezoidal and Simpson's rule cannot be applied



Points	Coordinates	
	X	Y
a	0	y_0
b	x_1	y_1
c	x_2	y_2
d	x_3	y_3
e	x_4	y_4
f	x_4	0
g	0	0
a	0	y_0



Sum of Products along the Solid line,

- $\sum P = (y_0 x_1 + y_1 x_2 + \ldots 0.0)$

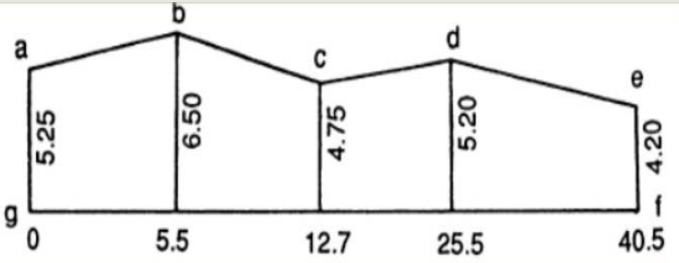
Sum of Products, along the dotted Line

- $\sum Q = (0.y_1 + x_1 y_2 + \ldots + 0.y_0)$
- **Required Area= ½ ($\sum P - \sum Q$)**

The following perpendicular offset were taken from a chain line to a hedge

Chainage (m) 0 – 5.5- 12.7- 25.5- 40.5

Offset (m) 5.25- 6.5- 4.75-5.2-4.2



Point	Coordinates	
	X	Y
a	0	5.25
b	5.5	6.50
c	12.7	4.75
d	25.5	5.20
e	40.5	4.20
f	40.5	0
g	0	0
a	0	5.25

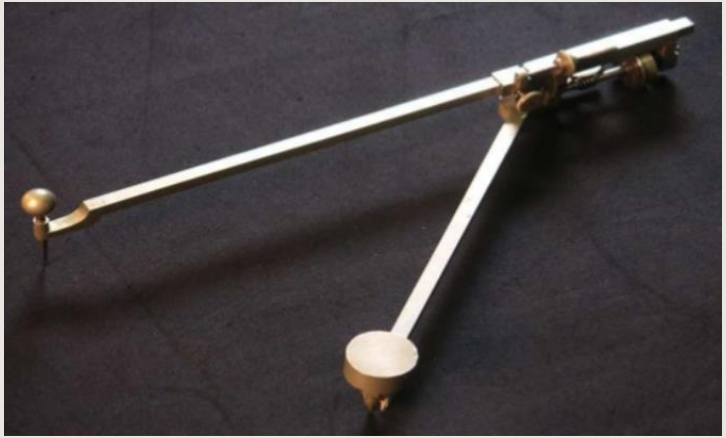
Then the coordinates are arranged in determinant form.

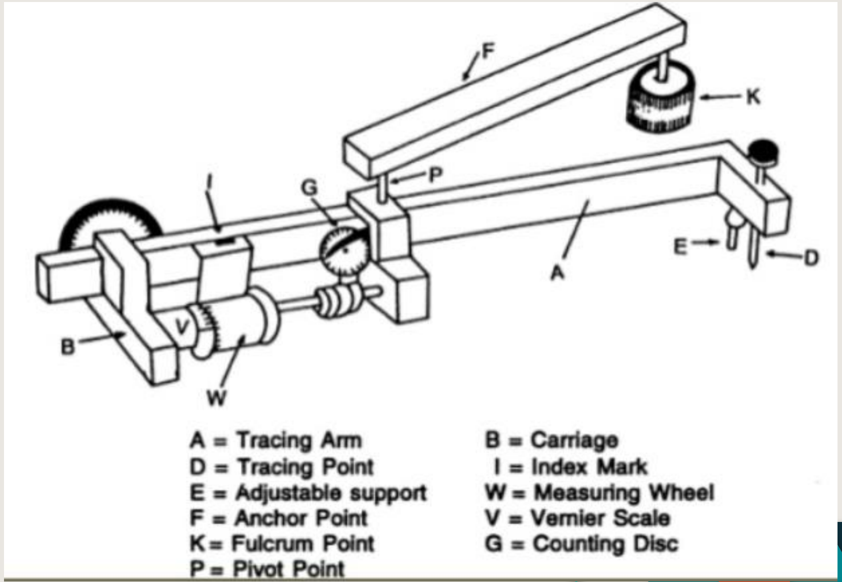
$$\begin{array}{cccccccc} a & b & c & d & e & f & g & a \\ \frac{5.25}{0} & \frac{6.50}{5.50} & \frac{4.75}{12.7} & \frac{5.20}{25.5} & \frac{4.20}{40.5} & \frac{0}{40.5} & \frac{0}{0} & \frac{5.25}{0} \end{array}$$

- Sum of products along the solid line,
- $\sum$ **P** = (5.25 x 5.5 + 6.5 x 12.7 + 4.75 x 25.5 + 5.2 x 40.5 + 4.2 x 40.5 + 0x 0 + 0 x 0).
- = 28.88 + 82.55 + 121.13 +210.6+ 170.1= **613.26 m²**
- Sum of Products along dotted line,
- $\sum$ **Q**= (0 x 6.5 + 5.5 x 4.75 + 12.7 x 5.2 + 25.5 x 4.2 + 40.5 x 0 + 40.5 x 0 + 0 x 5.25).
- = 26.13 +66.04 +107.10= **199.27 m²**
- Required Area= ½ ($\sum$ **P** - $\sum$ **Q**)
- = ½ (613.26-199.27) = **206.995 m²**

Instrumental Method

- The instrument used for computation of area from a plotted map is called planimeter
- Most common in use – Amsler Polar Planimeter





Planimeter – Construction details

- Arm A – tracing arm
- Tracing arm carries a tracing point D which is moved along the boundary
- An adjustable support E always keep the tracing point just clear of the surface
- Other arm – Polar arm or Anchor arm : pivots the instrument through a ball & socket arrangement
- Carriage consist of a measuring wheel W divided into 100 divisions and vernier V divided into 10 divisions
- Wheel geared into a counting disc graduated into 10 divisions, for 10 complete revolutions the disc shows a reading of one division

Procedure for finding out area

- A good starting point is marked on the boundary line
- By observing the disc, wheel and vernier, the initial reading (IR) is recorded
- TP is moved clockwise along the boundary
- The number of times the zero mark of the dial passes the index mark in clockwise/anticlockwise direction is observed
- Finally, by observing the disc, wheel, and vernier final reading (FR) is noted
- Area of figure, A = M (FR -IR$\pm 10N + C$)

where, M – Multiplier given in table

N – No of times the zero mark of dial crosses index mark

C – constant given in table

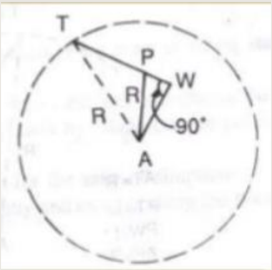
Table 1 Guiding Tables Supplied by Manufacturer

Scale	Vernier position on tracer bar (i.e. index mark)	Area for one revolution of measuring wheel (*M*)		Constant (*C*)
		Scale	Actual	
1 : 1	21.51	10 sq. inch	10 sq. inch	26.448
3/8″ = 1′ (1 : 32)	30.24	100 sq. ft	14.06 sq. inch	23.617
1/4″ = 1′ (1 : 48)	26.88	200 sq. ft	12.5 sq. inch	24.319
1/2″ = 1′ (1 : 24)	26.88	50 sq. ft	12.5 sq. inch	24.319

Table 2

Scale	Vernier position on tracer bar (i.e. index mark)	Area for one revolution of measuring wheel (*M*)		Constant (*C*)
		Scale	Actual	
1 : 1	33.33	100 cm^2	100 cm^2	23.254
1 : 200	33.33	0.4 m^2	—	23.254
1 : 400	20.83	1 m^2	—	27.133
1 : 500	26.67	2 m^2	—	24.637
1 : 1000	33.33	10 m^2	—	23.547

Planimeter – Zero Circle

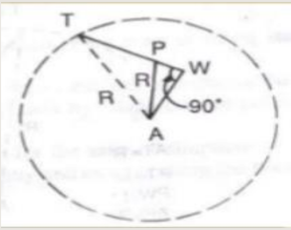


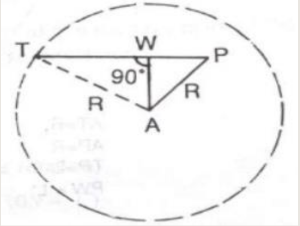
- When the tracing point is moved along a circle without rotation of wheel (i.e. when the wheel slides without any change in reading), the circle is known as Zero Circle
- It is obtained by moving the TP in such a way that the TP makes an angle 90 degrees with the anchor point
- The anchor point A is known as the centre of rotation and AT is known as radius of zero circle
- **When the anchor point is inside the figure, the area of zero circle is to be added to the area computed.**

Area of Zero Circle

Depending upon the position of anchor point, area of zero circle may be calculated as follows:

a. **When the wheel is outside the pivot and tracing point**





$$R = \sqrt{L^2 + 2LL_1 + R_1^2}$$

$$A = \pi R^2 = \pi(L^2 + 2LL_1 + R_1^2)$$

b. **When the wheel is between the pivot and tracing point**

$$A = \pi R^2 = \pi(L^2 - 2LL_1 + R_1^2)$$

where, L = length of tracing arm, i.e., distance between pivot and TP

L1 = distance between pivot and wheel

R1 = length of anchor arm, i.e., distance between pivot & anchor point

c. **By formulae**

Area of zero circle = M x C

Where, M = multiplier, value marked next to scale division = Area corresponding to one revolution

C = additive constant, value engraved on top of tracing arm just above scale division

Q4) Calculate the area of a figure from the following readings recorded by a Planimeter with the anchor point inside the figure.
IR = 8.277, FR = 2.256, M = 100 sq. cm and C = 23.521
It was observed that the zero mark on the dial passed the index once in the anticlockwise direction

Solution

Area of figure, A = M (FR -IR$\pm 10N + C$)

= 100 (2.256-8.277-10x1+23.521)

= 750 sq. cm

Q5) The following readings were obtained when the perimeter of a rectangle 15" x10" was traversed clockwise with the anchor point inside the rectangle & tracing arm set to the natural scale (M = 10 sq. inches). IR = 0.568, FR = 9.876. The zero of the counting disc passed the index mark twice in the reverse direction. Find the area of zero circle.

Solution

Area of rectangle = 15 x 10 = 150 sq. inch (i)

Measured area of rectangle, A = M (FR -IR$\pm 10N + C$)

= 10 (9.876-0.586 -10 x 2 +C) (ii)

Equating (i) and (ii)

10(-10.710+C) = 150

C = 25.710

Hence area of zero circle = MC

= 10 x 25.710 = 257.10 sq. in

Q6) The length of tracing arm of a Planimeter is 15.92 cm. The distance from the hinge to the anchor point is 16 cm. The diameter of the rim of the wheel is 2 cm. The wheel is placed outside (beyond the hinge from the tracing point) at a distance of 3 cm from the hinge. Calculate the area corresponding to one revolution of the wheel and the area of zero circle.

<u>Solution</u>

Area corresponding to one revolution = M = L x circumference of the rim

= 15.92 x π x 2 = 100.03 sq. cm

Area of zero circle, A = $\pi R^2 = \pi(L^2 + 2LL_1 + R_1^2)$

= π (15.92^2 + 2 x 15.92 x 3 + 16^2)

= 1900.555 sq.cm

VOLUMES

Volumes

- In many civil engineering projects, earthwork involve excavation, removal and dumping of earth, therefore it is required to make good estimate of volume of earthwork.
- Volume computation are also required to estimate reservoir capacity
- For volume computation, the sectional area of the cross-section which are taken to the longitudinal section during profile levelling are first calculated
- After calculating area, the volume can be computed using

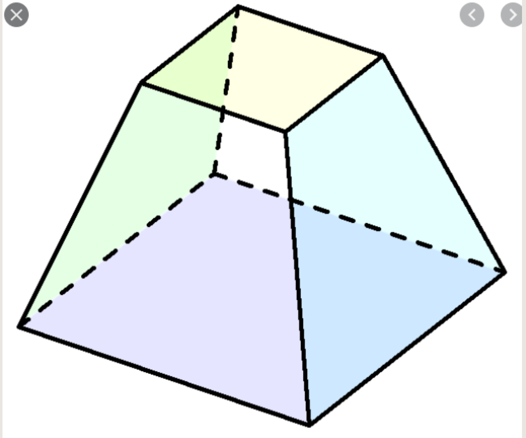
1. Trapezoidal rule
2. Prismoidal rule

Trapezoidal rule/Average End Area Rule

- Volume (Cutting or Filling)

$$= \frac{common\ distance}{2}(\text{First section area + Last section area +2 (sum of areas of other sections)})$$

Prismoidal Formula



- Prismoid : a solid bounded by end faces which are parallel
- Volume (Cutting or Filling)

$$= \frac{common\ distance}{3}(\text{First section area + Last section area +4 (sum of areas of even sections)+ 2 (sum of areas of odd sections)})$$

- Applicable only when the number of sections is an odd number

Prismoidal Correction

- The volume obtained by end area rule is not accurate
- Its accuracy can be increased by applying a correction called as prismoidal correction
- Prismoidal Correction (PC) = Volume by the end area formula – volume by the prismoidal formula

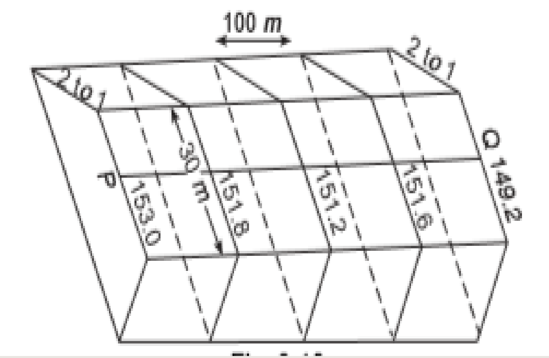
$$PC = \frac{d}{6} s(h_1 - h_2)^2$$

where,

h_1, h_2 = central heights of the two sections

s:1 = side slope, s horizontal to 1 vertical

Q1) A road embankment is 30 m wide at the top with side slopes of 2:1. the ground levels at 100 m along line PQ are as given:
P 153.0, 151.8, 151.2, 150.6, 149.2 Q
The formation level at P is 161.4 m with a uniform falling gradient of 1 in 50 from P to Q. Calculate the prismoidal formula the volume of earth work in cubic meters, assuming the ground to be level in cross section



Solution

Formation level at P = 161.4 m

Formation level at successive cross sections are obtained by deducting $\frac{1}{50}$ x 100 = 2 m from the level of preceding section

Formation level at P, 0 m = 161.4 m
Formation level at 100 m = 161.4 m – 2 m = 159.4 m
Formation level at 200 m = 157.4 m
Formation level at 300 m = 155.4 m
Formation level at 400 m = 153.4 m

Depth of embankment at various sections = Formation level – Ground level

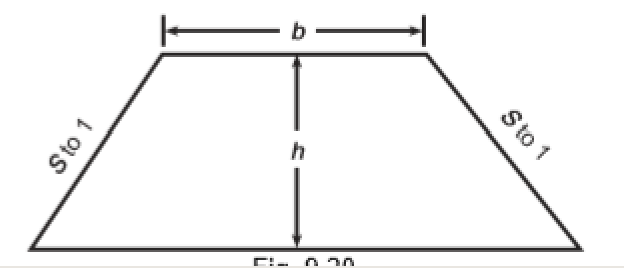
Depth at P, 0 m = 161.4 m – 153 m = 8.4 m

Depth at 100 m = 159.4 m – 151. 8 = 7.6 m

Depth at 200 m = 157.4 – 151.2 = 6.2 m

Depth at 300 m = 155.4 – 150.6 = 4.8 m

Depth at 400 m = 153.4 – 149.2 = 4.2 m



Areas

Area at cross section P, 0 m = A_1 = (b+sh)h

= (30+2 x 8.4) 8.4 = 393.12 sq. m

Area at 100 m, A_2 = 343. 52 sq m

Area at 200 m, A_3 = 262.88 sq m

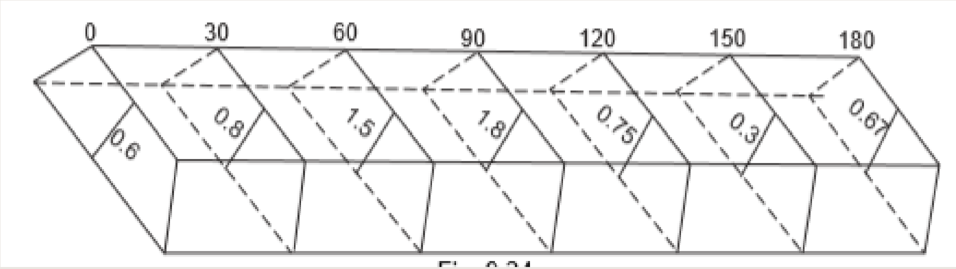
Area at 300 m, A_4 = 190.08 sq m

Area at 400 m, A_5 = 161.28 sq m

By prismoidal rule, Volume = $\frac{d}{3}(A_1+A_5+4(A_2+A_4)+2(A_3))$

= 107152 cubic meters

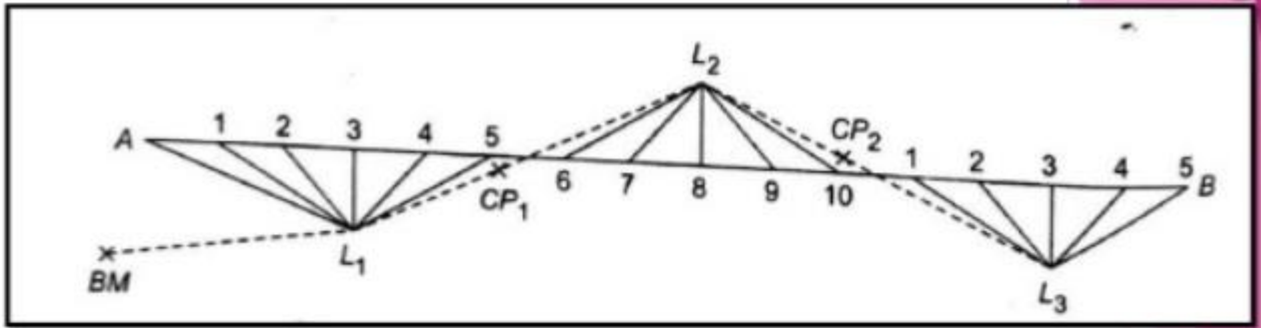
Q2) A railway embankment is 9 m wide at formation level, with side slope 2 to 1. Assume the ground to be level transversely, calculate the volume of the embankment in a length of 180 m, the centre heights at 30 m interval being 0.6, 0.8, 1.5, 1.8, 0.75, 0.3 and 0.67 m respectively. Use trapezoidal method



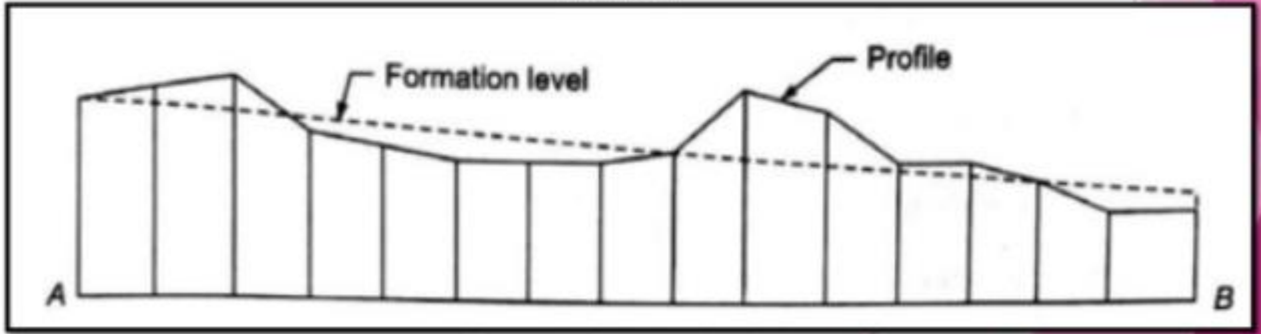
Solution

A1 = (b+sh)h = (9+2x0.6)0.6 = 6.12 sq. m
A2 = 8.48 sq. m
A3 = 18 sq. m
A4 = 22.68 sq. m
A5 = 7.875 sq. m
A6 = 2.88 sq. m
A7 = 6.928 sq. m

By trapezoidal rule, Volume = $\frac{d}{2}$ (A_1+A_7+2 (A_2+A_3+A_4+A_5+A_6))
= 1993.35 cu. m



Plan view



Drawn profile (sectional view)